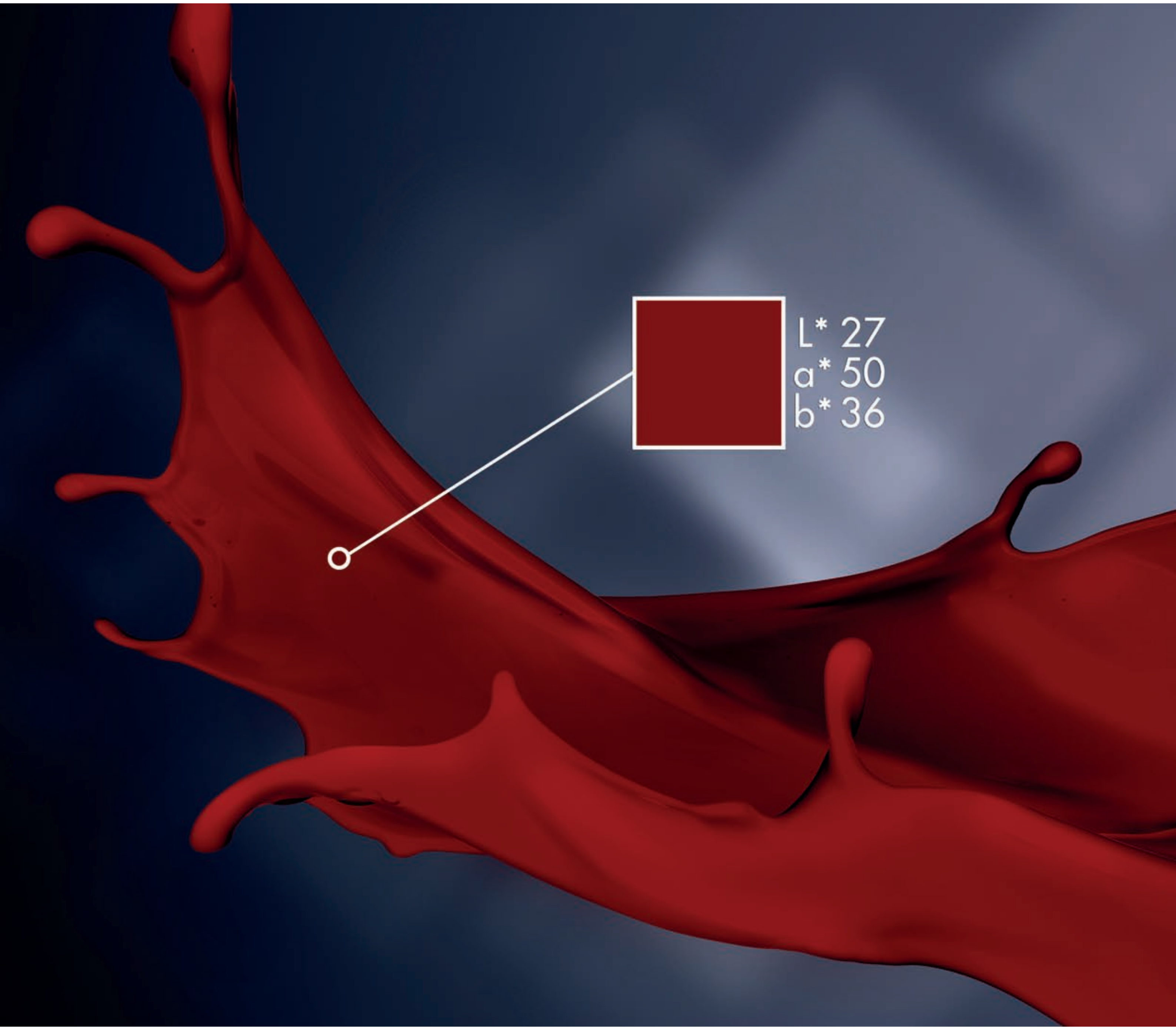
BOOK THREE OF COLOR MANAGEMENT



Color measurement - the CIE color space

datacolor

Chapter 9

Color measurement – the CIE 1931 system

Introduction

The measurement of a color is basically nothing other than standardised color vision, where the two factors, light and observer are standardised.

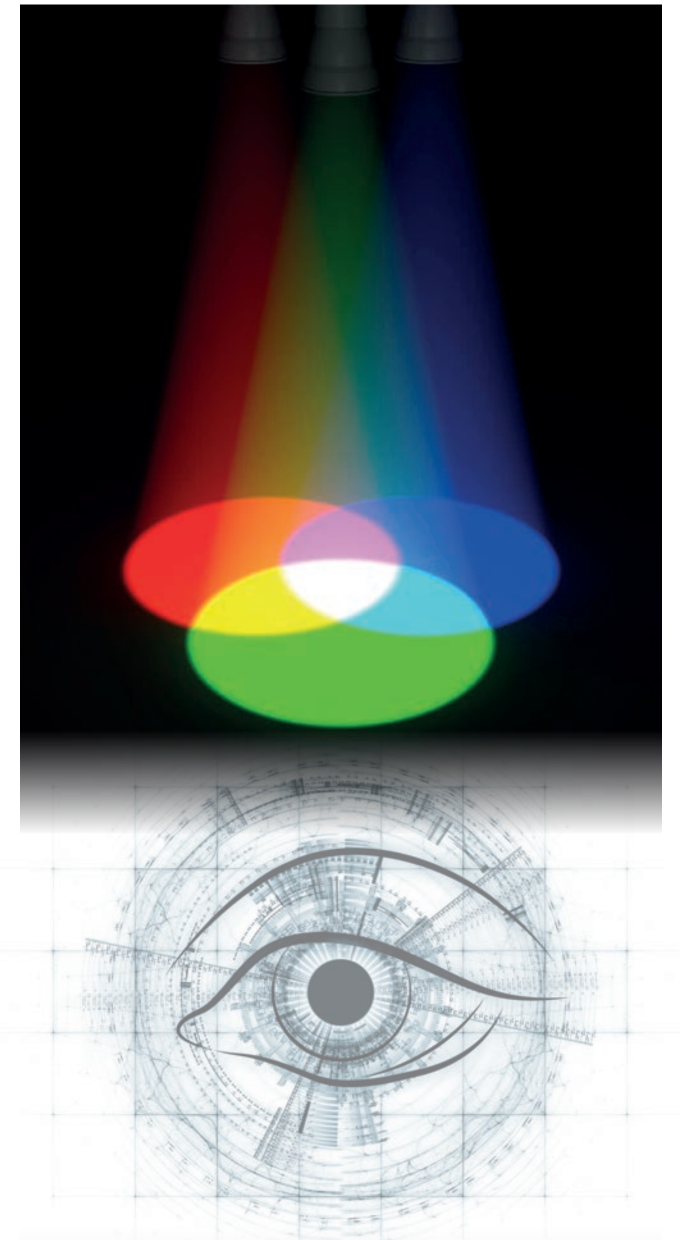
The scientific foundation for color measurement is based on the existence of 3 different groups of signals (primary color stimuli blue, green and red) that are passed on from the eye of the observer. The starting point for transfer into a standardised system is the sensitivity of the S, M and L cones. Today, the sensitivities, related to the wavelength, are known. As a basis for an international colorimetric system, the International Commission on Illumination, the CIE ("Commission Internationale de l'Éclairage") in 1931 defined three spectral colors as primary color stimuli – red R = 700.0 nm, green G = 546.1 nm and blue B = 435.8 nm.

The sensitivity of the cones, however, is also dependent on the angle of observation. Standardisation was implemented using the CIE standard observer concept. The standard observer, like the standard illuminant type, is a table of numerical values that represent an "average standard human observer", so the color perceptions are not specific to an individual observer.

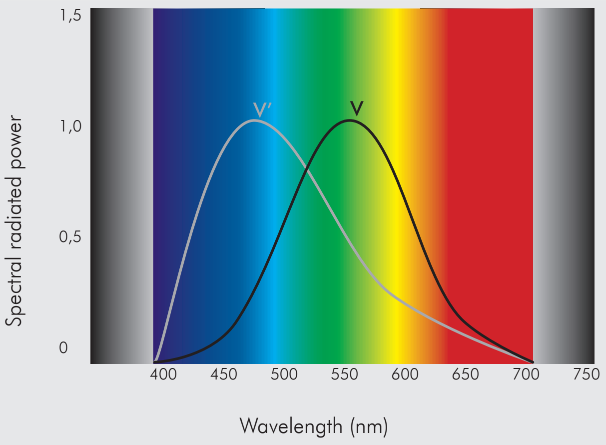
Luminous efficiency of the human eye – lightness

In the visible range of the electromagnetic spectrum (400 nm - 700 nm), the human eye perceives the same spectral radiances to have different lightnesses at different wavelengths. This spectral luminous efficiency of the eye was measured and standardised by the CIE for the standard observer. The V( ) curve applies for all photopic vision, in which the cones in the retina are active. The values for the luminous efficiency for photopic vision were drawn up in 1923 by the CIE, and adopted in 1924 to carry out colorimetric calculations.

The V´( ) curve applies for scotopic vision, in which the rods are the active receptors. The luminous efficiency values for scotopic vision were gathered into a standard by the CIE in 1951. In the luminous density range between photopic and scotopic vision – the mesopic range (twilight vision) – the spectral sensitivity curve shifts with a decreasing adaptive luminous density, even for shorter wavelengths.



Luminous efficiency of the eye



Luminous efficiency of the eye
Curve **V** *= photopic vision/day = (CIE 1924)*
Curve **V'** *= scotopic vision/night = (CIE 1951)*

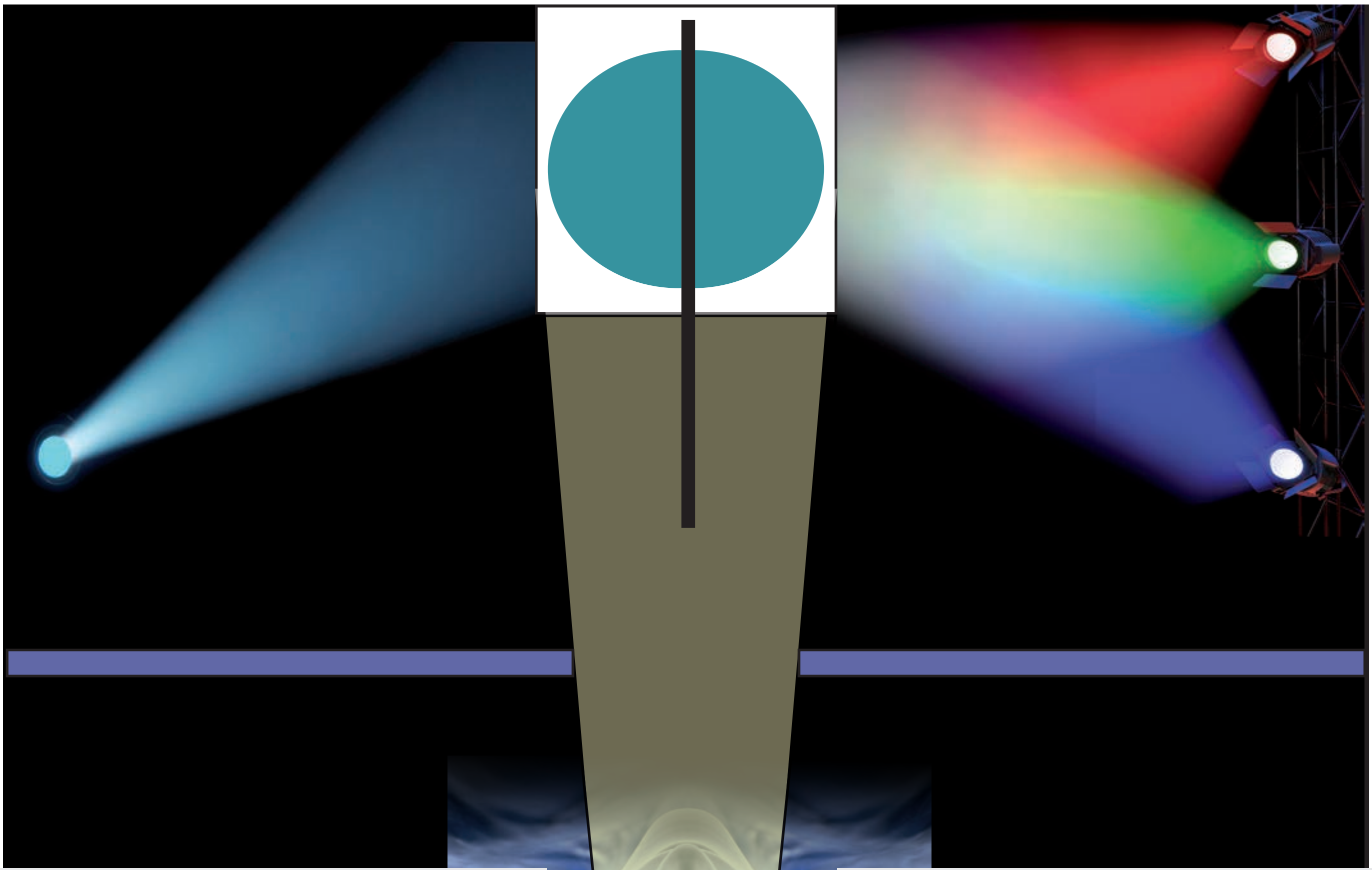
The human observer's perception of color stimulus

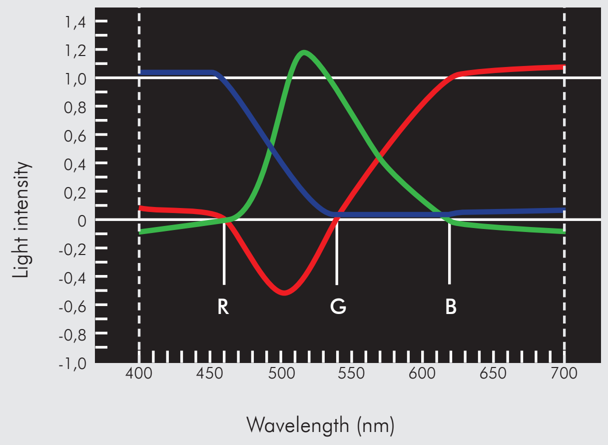
Experiments with normal-sighted human observers were carried out in order to define a "standard observer" as the basis for all colorimetric measures and calculations.

A divided screen was used in these experiments. On one side, one specific color was projected, and on the other side were three projectors in the light colors blue, green and red. The observer had to reconstruct the color impression of the first color by changing the lightness of the three light sources (three color theory). The radiation quantity was noted in a table for each change to the three primary light sources and each change to the experimental light at each wavelength.

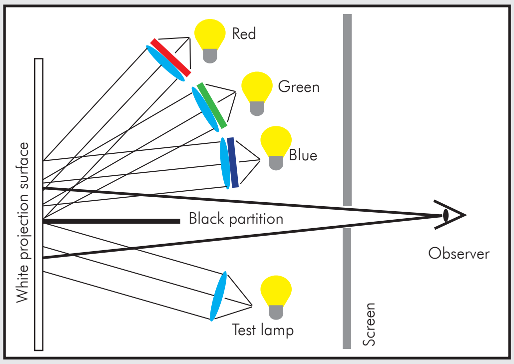
The entire range of color stimuli perceivable by people could be recorded in this way; color vision ability was recorded numerically.

The most significant experiments in determining the trichromatic perception of color stimulus of the eye were carried out in 1928 by W. D. Wright and 1931 by J. Guild. The trials by Wright and Guild were able to demonstrate that the numerical values differed slightly from each other because the primary light sources were slightly different. These experiments in additive color mixing corroborated Young's three-component theory.



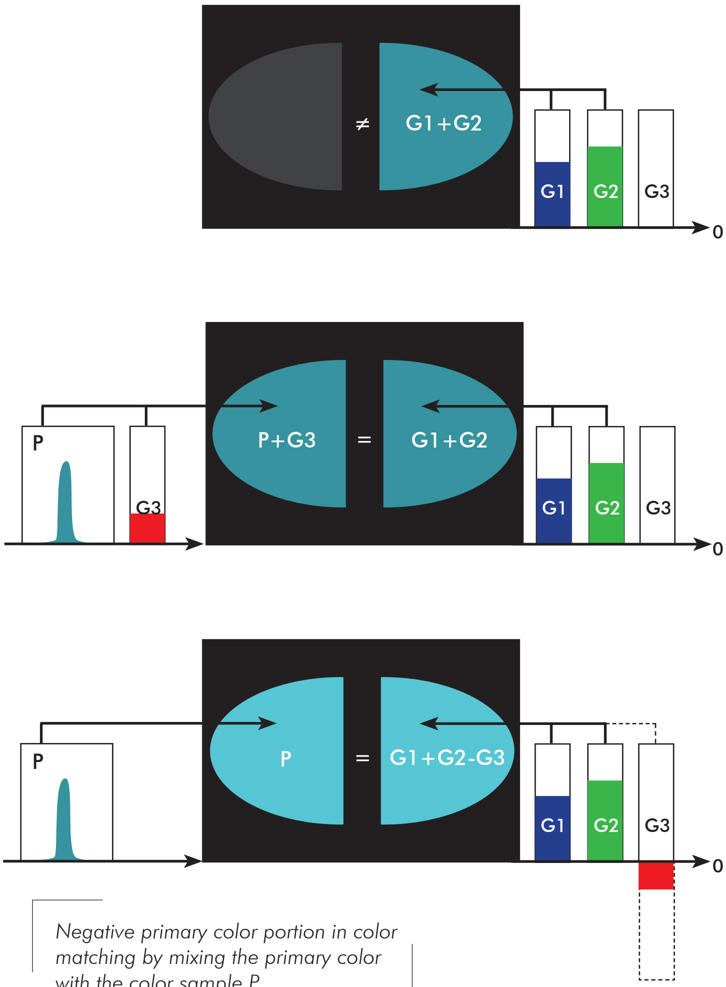


Experiment in the additive mixing of light radiation



The colorimetric 2° or small field CIE31 standard observer

In the experiments on additive color mixing, it was shown that not all real colors could be generated with the CIE's three RGB primary color stimuli. It was sometimes necessary to mix the color sample with one of the three primary colors to reach equivalence with the mixture from the two remaining primary colors. This means that certain colors can only be mixed from the three primary colors if one primary color contributes a "negative portion". For some spectral colors, therefore, the colorimetric values must be negative.



Negative primary color portion in color matching by mixing the primary color with the color sample P.

P + G3 = G1+ G2

is equivalent to

P = G1+ G2 - G3

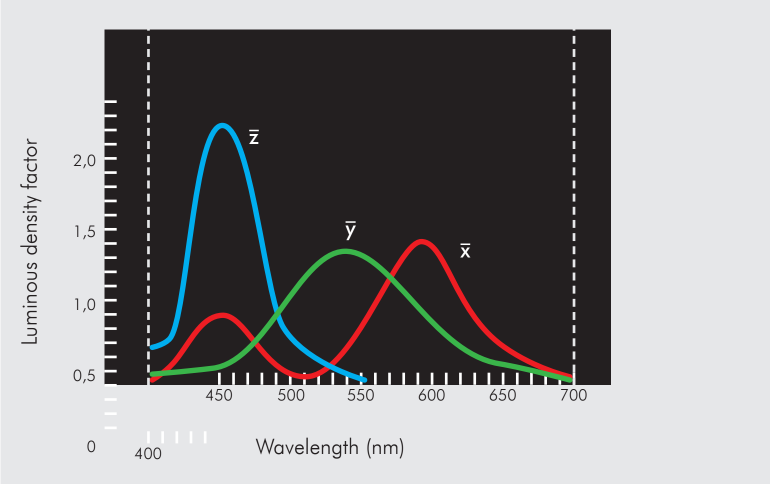
P ... Color sample, G1, G2, G3 ... three primary colors

On account of these restrictions, in 1931, the CIE defined three arbitrary imaginary measurement values (X, Y and Z) as primary color stimuli – selected from the point of view of easy colorimetric evaluation. All real colors can be represented by additive mixing using these three measurement values. These measurement values are called 'CIE standard tristimulus values', and the color space is called 'CIE XYZ color space'.

Transformation of the RGB primary color stimuli into the XYZ primary color stimuli was carried out with the following characteristics:

- Negative values in the equations were to be eliminated (negative values were extremely difficult to process electronically at that time)
- Definition of a new system with three 'imaginary' primary color stimuli $\bar{x}$, $\bar{y}$, $\bar{z}$, so the spectral locus falls into a triangle that is defined by these three primary color stimuli
- The function $\bar{y}$ was selected and calculated to correspond with the luminous efficiency function V ( ) (CIE 1924), therefore simplifying the calculations
- The function $\bar{z}$ was equated with zero for most of the visible spectral range, also to simplify the calculations
- The calculations were carried out for a light source with equal radiation as well as for the entire spectral range, so the areas of the $\bar{x}$, $\bar{y}$, $\bar{z}$-functions are the same.

The resulting functions are called CIE-$\bar{x}$, $\bar{y}$, $\bar{z}$-color-matching functions. They are not real functions in the proper sense; they represent the average standard observer.



Color-matching functions $\bar{x}$, $\bar{y}$, $\bar{z}$ of the 2° standard observer (CIE31)

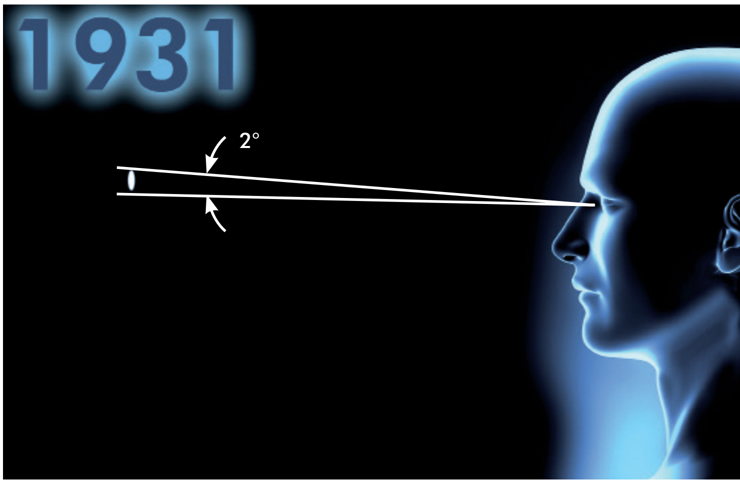
The colorimetric 10° or large field CIE64 standard observer

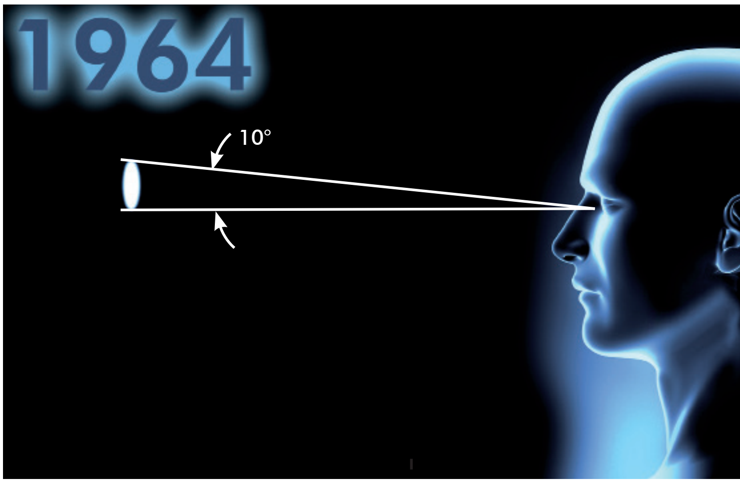
To allow human perception to be incorporated in a controlled way into a measurement result, it was necessary to define a standard for human vision. This standardised vision was defined by the CIE standard observers.

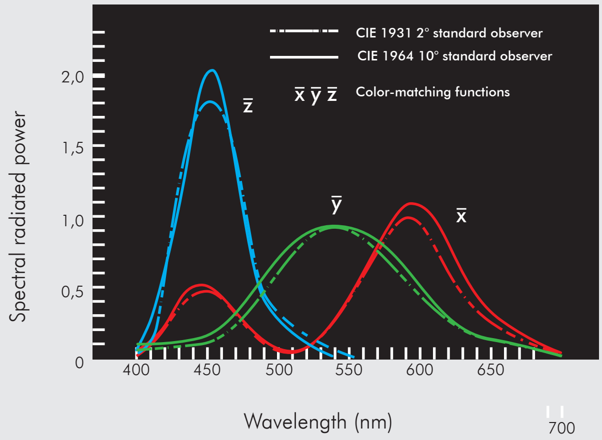
The CIE31 standard observer or 2° standard observer is traced back to an experiment to determine the average color perception of the human observer, For this, it was considered that humans perceive colors most precisely when they encounter the area in the eye where vision is sharpest (fovea, yellow spot). With a normal viewing distance from a color sample, this region deviates by about 2° from the optical axis of the eye. From this, it was determined that the angle at which the standard observer sees should be exactly 2°. This corresponds to a field of vision the size of a 1 euro coin that you hold in front of you with your arm outstretched.

The normal field of vision of human perception, however, is greater than this 2° area. Jacobsen (1948) and Judd (1949) were also able to demonstrate that the colorimetric calculations based on the 2° angle did not correspond very well to the actual observations in the range of the short wavelengths (especially for violet). As a result, the CIE proposed another standard observation angle in 1960 – the 10° standard observer. This corresponds to a field of vision the size of an A4 page at a standard viewing distance of 30 cm. The color-matching functions $\bar{x}_{10}$, $\bar{y}_{10}$, $\bar{z}_{10}$ of this new standard observer were then ultimately specified by the CIE as standard in 1964.

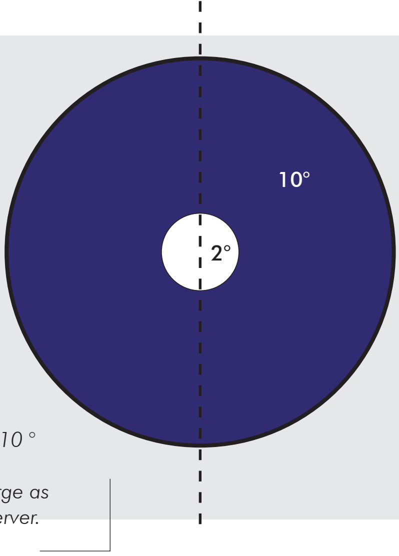
The field of vision of 2 ° standard observer corresponds to the size of a coin of 1 Euro that you hold with an outstretched arm in front of you.







Color-matching functions $\bar{x}$, $\bar{y}$, $\bar{z}$ of the 10° standard observer (CIE 1964)



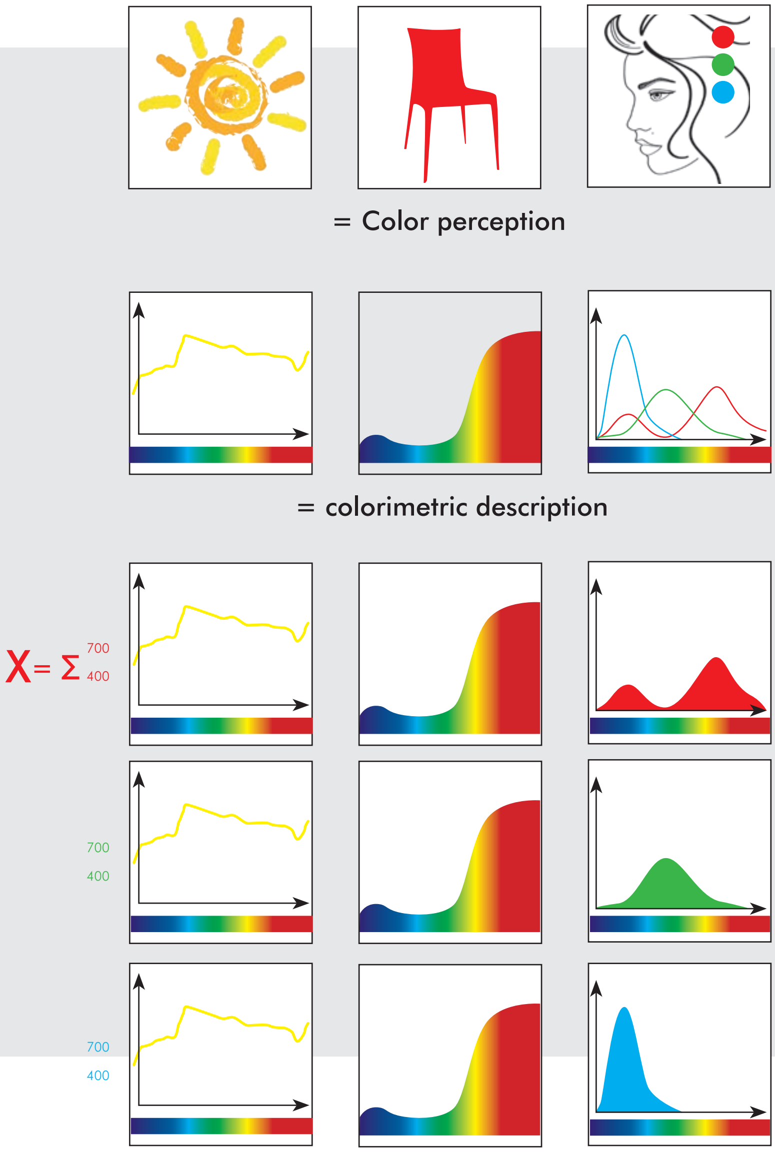
The field of vision of 10 ° standard observer is about 27 times as large as the 2 ° standard observer.

The chromaticity diagram according to the CIE standard colorimetric system CIE31

Using the standard color-matching functions $\bar{x}$, $\bar{y}$, $\bar{z}$ of the standard observer, the spectral curve can be converted into 3 values – the standard tristimulus values X, Y and Z. By means of these standard tristimulus values, the color of an object or a light source can be determined with three measurement values.

Example: Calculation of the standard tristimulus value X of a color specification.

For each wavelength of the visible spectral range, the value of the color-matching function **x** is multiplied by the value of the spectral radiated power S of a standard illuminant type for the same wavelength. This calculation is carried out for each selected wavelength increment (d) in the entire spectral range (400 nm – 700 nm). The total of these calculated products for all wavelengths (∑ of 400 – 700 nm) is then calculated.



For the precise classification of color, we need:

1. The radiation distribution for the illuminant type (E)
2. The wavelength-dependent physical reflectance/reflection coefficient of the object (R)
3. The color specifications for the observer/the color-matching function of the standard observer $\bar{x}$

For the calculation of the color specification of a colored object, the radiation S (λ) is equated with the product E (λ) • R (λ) for each wavelength, that is, the radiation of the light source E (λ), which illuminates an object, is reduced by the percentage of the reflection coefficient of this object, and this is the case for each wavelength increment (Δλ).

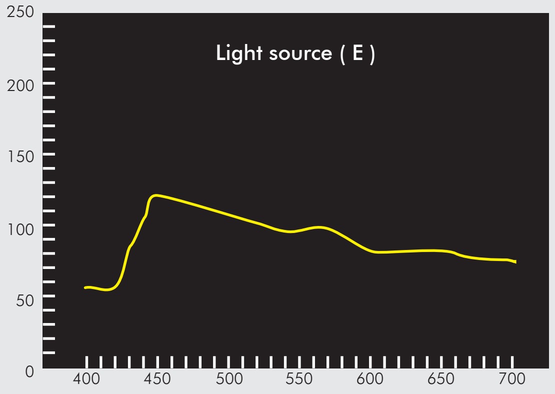
Therefore, the mathematical formula for the standard tristimulus value X of the colored object is:

$$X = \sum_{400}^{700} E(\lambda) \bullet R(\lambda) \bullet \bar{x}(\lambda) \bullet \Delta\lambda$$

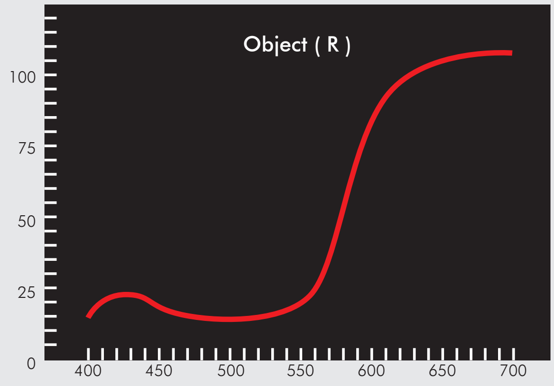
where

dE = the radiation of the light source (illuminant type)
R = the reflection coefficient of the object
$\bar{x}$ = the color-matching function of the standard observer
λ = symbol for the wavelength; if (λ) comes after another symbol, this means that it is wavelength-dependent

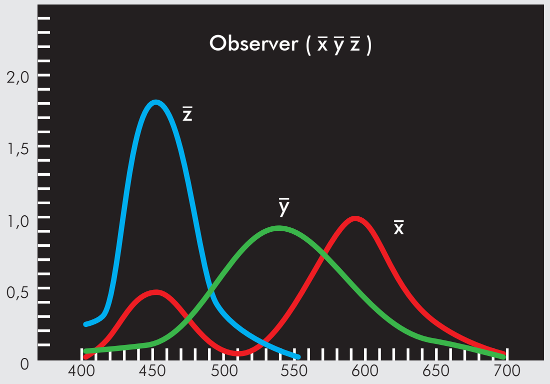
Y and Z are calculated in the same way.



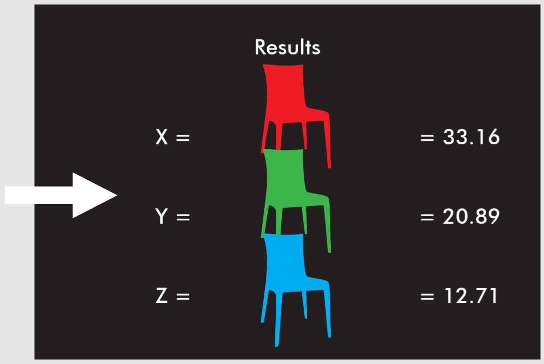
Standard illuminant type CIE D65 – spectral radiated power



Red sample – reflection curve



2° CIE31 standard observer – color-matching functions



CIE color specifications X – Y –Z = 3 numerical values

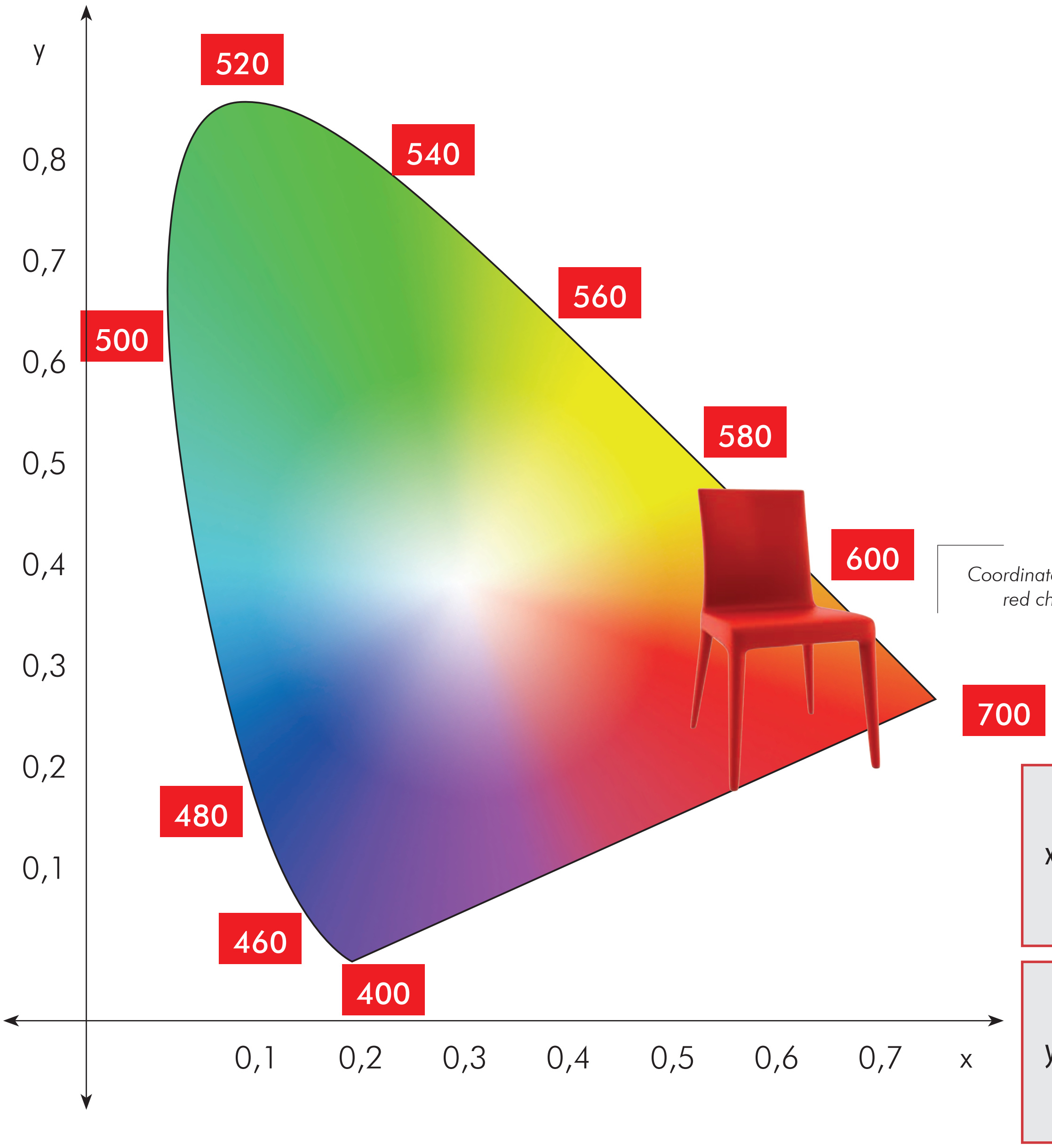
Spectral radiated power (E) x reflection coefficients (R) x color-matching functions ($\bar{x}$,$\bar{y}$,$\bar{z}$) =3 color values (X,Y,Z)

The calculation principle for the standard color values XYZ

The chromaticity diagram according to the CIE standard colorimetric system CIE31

Using the standard tristimulus values XYZ of the CIE31 standard colorimetric system, a color can be determined with high precision. However, correlation with the visual evaluation is unfortunately often very difficult. Even if the standard tristimulus value Y corresponds relatively well to the sense of luminance, the X and Z values can only with great difficulty be approximated to the criteria for hue and chroma in the visual perception of colors.

In order to be able to more clearly represent the three-dimensional color space perceived by the observer (according to chromaticity), the two-dimensional CIE standard chromaticity diagram was developed. This can be used to determine the color specifications separately from the lightness. The CIE introduced the standard chromaticity coordinates x, y, z for this, whereby x and y are used to determine the chromaticity. "Small x" is the relatively red coordinate and, accordingly, "small y" is the relatively green coordinate (z being unnecessary, since $z = 1 - x - y$). For the shoe in our example, the chromaticity coordinates appear as follows: $x = 0.4967$ and $y = 0.3129$ for standard illuminant D65 and a 2° standard observer.



Coordinates of the color shade of the red chair in the CIE color triangle

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

For graphical representation, the CIE proposed a coordinate system, with x as the abscissa and y as the ordinate. The chromaticity coordinates of the pure colors in the visible spectral range form a concave curve shaped like the "sole of a shoe". This is called the spectral locus. In the inner area of the "sole of the shoe", also called the color triangle, all possible colors are represented (in light). Each color point within this area has a different chromaticity. The green and blue shades are in the upper area of the color triangle, the violet shades on the lower left-hand side, and the red shades on the lower right. The straight connection line between violet and red is called the line of purples (purple is not a spectral color!). The area enclosed contains the color points of all of the real chromaticities.

In the centre of this area is the neutral achromatic point (x=0.333, y=0.333) of a light source with equal-energy radiation, also called the white point. The white point changes according to the illuminant type used, as each illuminant type has a different spectral composition. Standard illuminant A (light from a lightbulb) is in an area much more yellow/orange than the other standard illuminants. Standard illuminant D65 (daylight) is whiter and is close to the central area.

For simpler determination and classification of a color in the CIE31 standard chromaticity diagram, the equal-hue wavelength and the chroma of a color can be defined instead of the standard chromaticity coordinates. This method allows a color to be defined according to hue and chroma, as in visual classification. This is also the advantage of this method. The equal-hue wavelength is the wavelength that corresponds to the additive color mixing for the required color. It describes the hue of the pure color point. The chroma is the percentage proportion of the pure color in the mixture. The highest chroma is equal to 1. This corresponds to the pure color. A chroma of 0 corresponds to the color of the illuminant type (white light). The chroma is at its highest on the spectral locus and its lowest at the central achromatic point.

Conclusion – notes

In summary, so far it can be said that the color of an object, such as the red sample, can be precisely determined using the CIE31 standard colorimetric system, by means of the three measurement values X, Y and Z, and taking into account the standard illuminant type and the CIE31 standard observer.

The CIE31 standard colorimetric system forms the scientific basis of modern color measurement.

All works and research on the development of new colorimetric formulae from 1936 up until today have been based on this system, and although it allowed the high precision determination of a color by means of the three measurement values, it has repeatedly been the subject of numerous studies and improvements. You can learn about some of these in the following chapters.

Chapter 10

The color spaces

General

Tristimulus notation made possible the first map of color space, the CIE Chromaticity Diagram. Developing a map was a decisive moment in the science of colorimetry. When you can locate objects on a map, you can measure the distance between them.

These distances represent (however imperfectly) the color differences between the samples. The CIE Chromaticity Diagram was the first tool widely used to express visual differences using numbers.

It remains the basis for the ongoing efforts to develop and refine the maps of color space and colorimetric calculations.

Today we use numbers to specify colors, describe color differences, set color tolerances and evaluate the consistency and acceptability of colored products. Most notably we now see better agreement between the numerical description of color differences and what we see visually.

The CIE31 standard colorimetric system, on the other hand, is based on the physical characteristics of light. The CIE system aims to achieve uniformity and standardisation of the light colors and sources as well as the body colors. The perceptual uniformity of the color scale is not considered here. MacAdam was able to demonstrate this non-uniformity of the CIE31 color space with his experiments on the visual perception of colors (1942). With illumination conditions constant, an observer viewed 2 colors – one color was fixed, and the second color had to be adjusted by the observer so that it was identical to the test color. This experiment was carried out with 25 different test colors from the CIE31 diagram. The adjusted colors all lay in an ellipse around the original test colors, whereby the form and orientation of the ellipses were very different, depending on color.

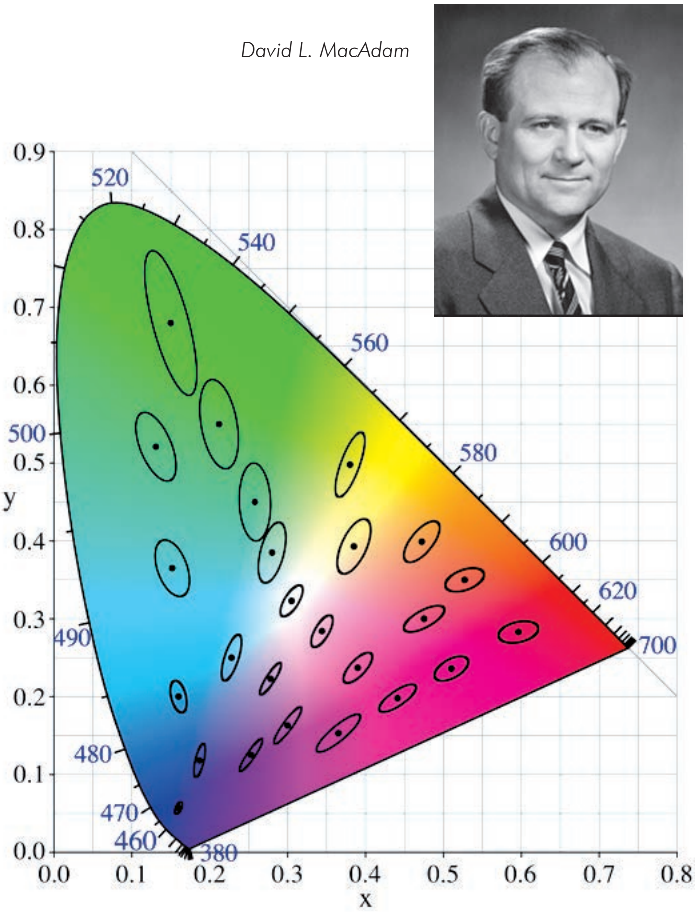
History – development from 1905 to 1976

At the beginning of the 20th century, Albert Henry Munsell developed a color system on a scientific basis. He determined colors according to the measurable characteristics of color shade, lightness, and chroma, arranging them three-dimensionally. In 1905, he published "A Color Notation", which describes the system. The first color atlas showing the three-dimensional color space from different perspectives appeared in 1915.

The MUNSELL color atlas was oriented to the human observer's sense of color, featuring

- An optically balanced design (uniform color space) and
- A method for the reciprocal determination of colors, where each color can take up only one single place.

David L. MacAdam



CIE color diagram with MacAdam-Ellipsen

This prompted the CIE to develop mathematical transformations for the CIE31 color space, in order to maintain a uniform color space.

The CIE recommended two new systems in 1976 – the CIELuv and CIELAB color spaces. To make a distinction from other systems (especially the Hunter system), a * was added to all of the parameters used (for example, L*, a*, b*).

The CIELuv system is preferably used for additive color mixing, e.g. light color measurement in scanners and monitors. The CIELAB system is limited to the examination of body colors.

As the CIELAB system is the most commonly used in color measurement applications, we will examine this system more closely.

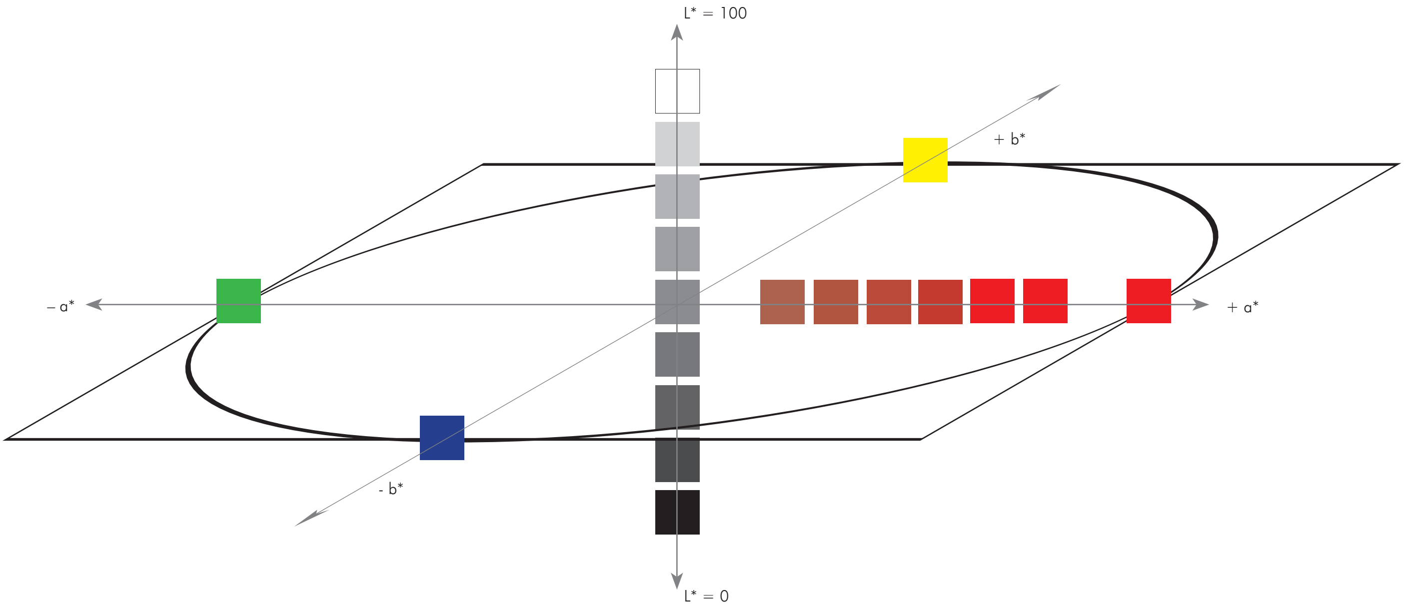
The CIELAB color space – definitions and characteristics

The 1976 CIE color space, also called CIELAB color space, is based on a nonlinear transformation of the 1931 CIE tristimulus coordinates X, Y, Z. The development of this color space had two objectives:

- Uniformity. The calculated distances between samples must uniformly correlate to visual differences between samples
- Simplicity. It should offer a simple means for the user to interpret the data.

The CIELAB color space, with some conditions, is a uniform scale and device-independent. Each perceivable color in color space is defined with the color point that has coordinates {L*, a*, b*}. Here, in application of the opponent color theory, green and red lie against the a* axis. The b* axis corresponds to the opponent colors blue and yellow. The L* axis stands perpendicular on this plane and represents the lightness. The L* axis can also be referred to as the neutral grey axis, as its endpoints are black (L=0) and white (L=100), the values in between on this axis are achromatic shades of grey.

The CIELAB color space: the determination methods.



The formula for the transformation and calculation of the CIELAB color space based on XYZ is as follows:

Color values: : L* a* b*

$$L^* = 116\ f\left(\frac{Y}{Y_n}\right) - 16 \qquad a^* = 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \qquad b^* = 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right]$$

where

- If $\frac{X}{X_n} > \left(\frac{6}{29}\right)^3,\quad f\left(\frac{X}{X_n}\right) = \sqrt[3]{\frac{X}{X_n}}$, else $f\left(\frac{X}{X_n}\right) = \frac{841}{108}\left(\frac{X}{X_n}\right) + \frac{4}{29}$
- If $\frac{Y}{Y_n} > \left(\frac{6}{29}\right)^3,\quad f\left(\frac{Y}{Y_n}\right)\ \sqrt[3]{\frac{Y}{Y_n}}$, else $f\left(\frac{Y}{Y_n}\right) = \frac{841}{108}\left(\frac{Y}{Y_n}\right) + \frac{4}{29}$
- If $\frac{Z}{Z_n} > \left(\frac{6}{29}\right)^3,\quad f\left(\frac{Z}{Z_n}\right)\ \sqrt[3]{\frac{Z}{Z_n}}$, else $f\left(\frac{Z}{Z_n}\right) = \frac{841}{108}\left(\frac{Z}{Z_n}\right) + \frac{4}{29}$

The values X_n, Y_n, Z_n are the color values of the absolute white (ideally an achromatic color stimulus) of a body color for the particular CIE standard illuminant type (e.g. D65 or A). Under these conditions, X_n, Y_n, Z_n are color values for the standard illuminant type, where Y_n is equal to 100.

E.g. for D65/10°:

X_n = 94.81

Y_n = 100.00

Z_n = 107.304.

To describe the color distances using the correlation factors of lightness, colorfulness (chroma) and hue, the following defined rules can be used:

- **Lightness CIELAB: L***
  The parameter L* is defined above
- **Chroma – Colourfullness CIELAB: C***
  The parameter C* is defined by the following relation:

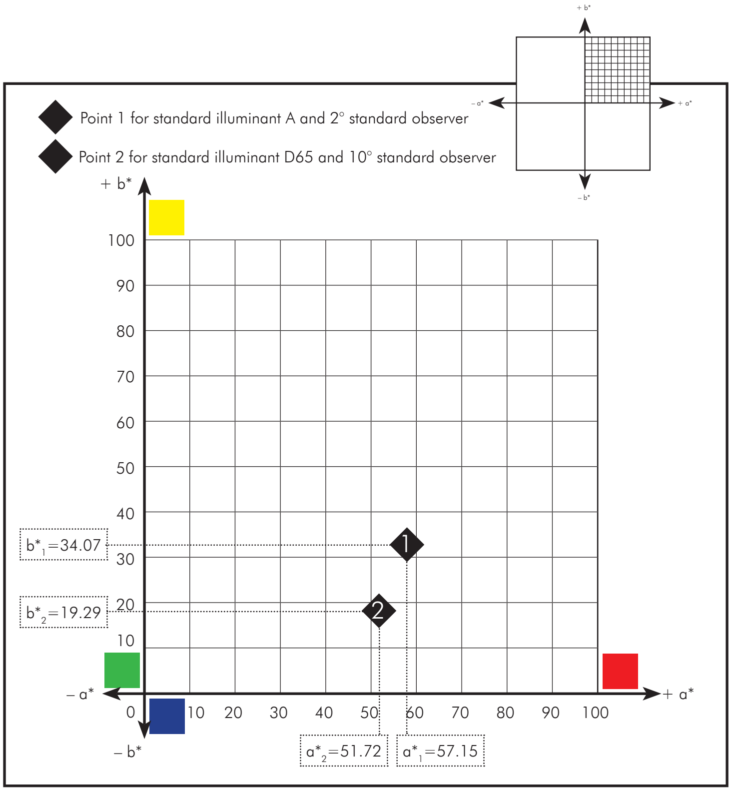
  $$C^* = \sqrt{(a^{*2} + b^{*2})}$$

- **Hue angle CIELAB: h**
  The parameter h is defined by the following relation:

  $$h = \text{arc tg}\left(\frac{b^*}{a^*}\right)$$

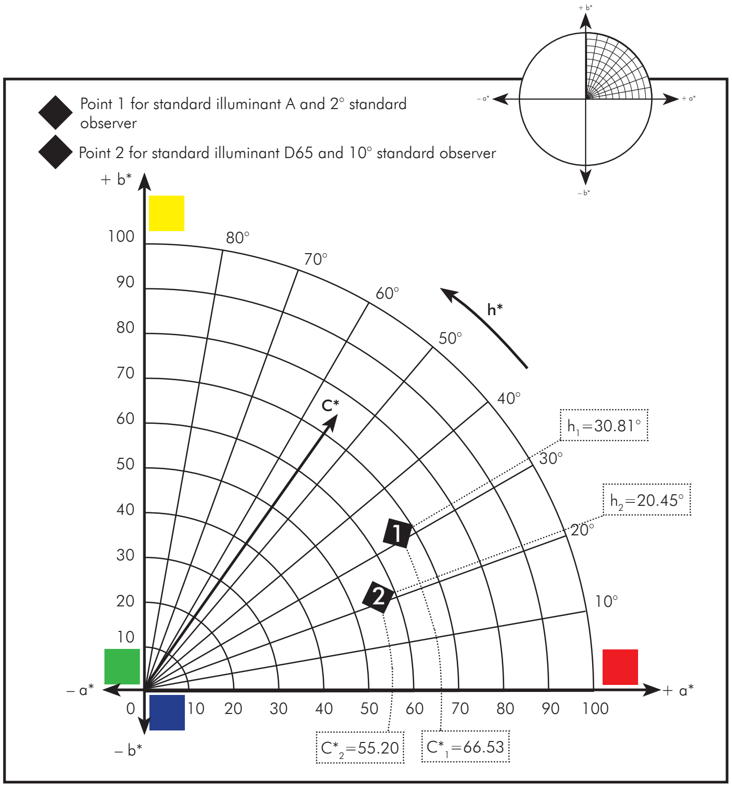
Furthermore, the CIELAB color space has the characteristics of an Euclidean space. Each point can be described by:

- **its perpendicularly plotted coordinates L*, a* and b*, where**
  - L* represents the lightness
  - a* represents the red/green color specification
  - b* represents the yellow/blue color specification



Position of the color points in perpendicularly plotted coordinates L, a* and b* of the CIELAB system*

- **Or by its cylindrically plotted coordinates L*, C* and h, where**
  - L* still represents the lightness
  - C* represents the colorfulness or the chroma
  - h represents the hue angle or the shade of the color.



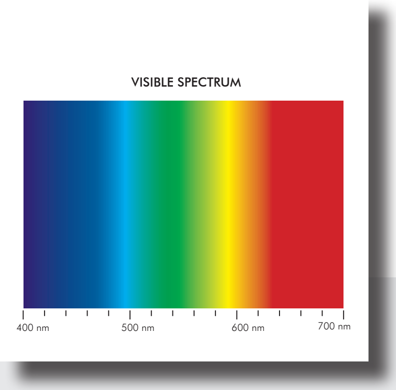
Position of the color points in cylindrically plotted coordinates L, C* and h of the CIELAB system*

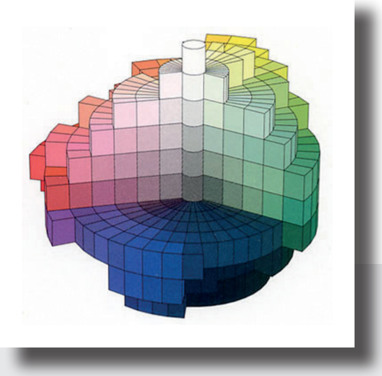
As a result of the transformation, there is no chromaticity diagram for the CIELAB color space. In the color planes defined by the values a*, b* or C*, h, the colors cannot be added any more.

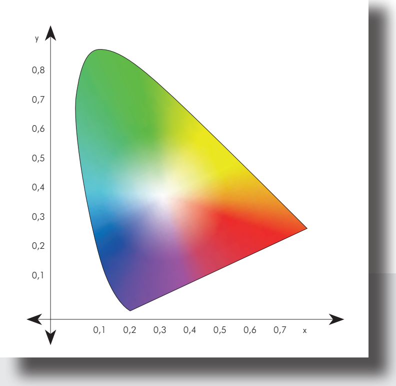
The CIELAB color space is roughly (not absolutely) structured according to perception; statistically, it corresponds to human visual color perception. Therefore, it is not strictly uniform for color evaluation in terms of the psychology of perception, but it simplifies the interpretation of a color point and of colorimetric deviations.

From color to color measurement

The previous chapters dealt with the development of color measurement, from visual evaluation to the determination of a color by means of the standardised color systems CIE31 and CIELAB 1976, and always considering individual or isolated colors. The following chapters will deal with the evaluation of distances between two or more colors and the acceptability of colors.

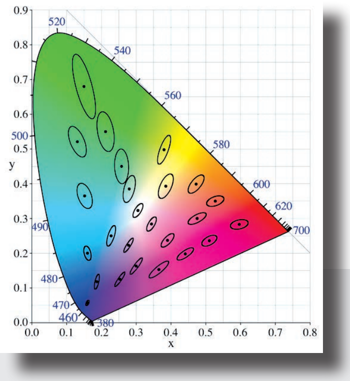


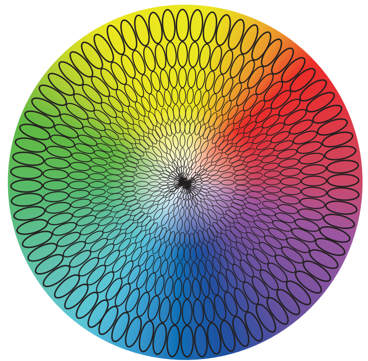




Prior to Color measurement		
Always	1905-1915	
Language	Munsell-Atlas	CIE31
"red" "vibrant" "light"	Sample 2.5 R 5/12	X = 33.16 Y = 20.89 Z = 12.71 for D65 / 2°
Limited vocabulary	Determination that can be compared	First calculation
Determined by people		
SUBJECTIVE		

From color … to color measurement
From technical language … to the 1931 CIE color space … and to the 1976 CIELAB color space
From the subject … via the object to the measurement





With Color measurement		
1931	1976	
Color triangle	CIELAB color space	
x = 0.4967 y = 0.3129 Dominant L_1 = 628 nm Luminous density = 46.9 % for D65 / 2°	L* = 52.15 a* = +51.72 b* = +19.29 for D65 / 10°	L* = 52.15 C* = 55.20 h = 20.45° for D65 / 10°
First objective determination	Determination via perpendicularly plotted coordinates	Determination via cylindrically plotted coordinates
Colorimetric calculation		
OBJECTIVE		

List of references

- Farbe sehen, Corinna Watschke, 01.2009 [www.planet-wissen.de],
- Farbmanagement in der Digitalfotografie (ISBN 3-8266-1645-6), 2006, Redline GmbH, Heidelberg
- Beschreibung und Ordnung von Farben, Farbmetrik, Farbmodelle, DMA Digital Media for Artists – Archiv 2006-2011, Kunstuniversität Linz, Gerhard Funk
- Messen – Kontrollieren – Rezeptieren, Dr. Ludwig Gall [www.farbmetrik-gall.de]
- Farbabstandsformeln, 2012, Fogra Forschungsgesellschaft Druck e.V. [www.fogra.org]
- Wikipedia, diverse Artikel über Farbe und Farbmessung [http://de.wikipedia.org/wiki/Farbe]
- Verschiedene Darstellungen von Farbmodellen und Farbräumen [http://www.chemie-schule.de/chemieWiki_120]
- Praktische Farbmessung, Anni Berger-Schunn, 2. überarbeitete Auflage, 1994, Muster-Schmidt Verlag, Göttingen – Zürich
- Farbabstandsformeln in der Praxis, SIP 01.2011
- Schläpfer, K.: Farbmetrik in der grafischen Industrie, 3. Aufl. St. Gallen; UGRA 2002 (Tabelle S. 48)

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